For configuration 4a, for $R_S \le (\xi_1 - W_2/L)$, no shadowing effect will be there, the view factor expression reduces to the form available in literature (configuration 4 in Ref. 1).

Conclusions

Closed-form solutions are presented for the view factors between a coaxial differential element and a finite area, when an intervening finite area at an arbitrary position is present in between. Four totally different combinations of geometrics are considered and the finite areas could be either a circular disk or a rectangle. Depending on the position and dimensions of the intervening surface, analytical expressions have been presented for a total of 14 configurations. When the intervening surface is not present and/or when it has particular dimensions and positions, the expressions presented reduce to the form available in literature.

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One-Dimensional Analysis of Hollow Conical Radiating Fin

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Nomenclature

H, R, t	=	height, radius, and thickness of the fin, m
J	=	radiosity, W/m^2
k	=	thermal conductivity, W/m K
Q	=	rate of heat loss, W
T	=	temperature, K
ε	=	emissivity
θ	=	fin angle, rad
ρ	=	density, kg/m^3
σ	=	Stefan–Boltzman constant

Subscripts

e	=	corresponds to space
I, O	=	correspond to inside and outside fin surfaces
im	=	improvement per unit mass
UB	=	corresponds to unfinned isothermal base

Introduction

BECAUSE the mass is at a premium on spacecraft, several researchers[1–6] have attempted to optimize the finned space radiators used for waste heat rejection. Bhise et al.[7] investigated a corrugated structure in this regard. Srinivasan and Katte[8] proposed a grooved radiator with higher heat loss per unit mass compared to the flat radiator. A literature review shows that there are only a few attempts to modify the configuration while optimizing the radiating fins. Presently, a hollow conical configuration for radiating fin is proposed to augment the heat loss per unit mass. A one-dimensional analysis of such a fin is carried out. Effects of various parameters are studied and correlations are presented for optimum parameters.

Analysis

The hollow conical fin (Fig. 1) radiates heat from inside, outside, and tip surfaces in addition to the base surface, which is assumed to be maintained at T_B. The assumptions are as follows: the heat conduction is one dimensional along the axis, all of the surfaces are diffuse and gray, and the space is a black surface at T_e. Radiosity-irradiation method is used to account for the fin-base interaction and the interaction among the fin enclosure itself. As a conservative approach to account for the fin-base interaction, R_B is taken as twice R_T for $\theta = 10$ deg.

For the differential element (Fig. 1), the energy balance equation can be shown to be

$$\frac{\mathrm{d}}{\mathrm{d}x}\left(r\frac{\mathrm{d}T}{\mathrm{d}x}\right) + \frac{r}{kt\sin\theta}\frac{\varepsilon}{(1-\varepsilon)}\left(\sigma T^4 - J_I\right)\mathrm{d}x + \frac{r}{kt\sin\theta}\frac{\varepsilon}{(1-\varepsilon)}\left(\sigma T^4 - J_O\right)\mathrm{d}x = 0 \tag{1}$$

with boundary conditions

$$T(x=0) = T_B \qquad \text{and} \qquad \left.\frac{\mathrm{d}T}{\mathrm{d}x}\right|_{(x=H)} = \frac{-\sigma\varepsilon}{k}\left(T^4 - T_e^4\right)\Big|_{(x=H)}$$

The view factors for inside and outside surfaces are calculated using expressions for parallel coaxial disks of unequal radii (configuration C-41),[9] annular disk to coaxial truncated cone (configuration C-49), and view factor algebra. Because the temperatures and radiosities are coupled, an iterative method, in general, is used. Based on the assumed temperatures, the radiosities are calculated using the Gauss–Seidel technique for each enclosure. Using these radiosities, the nodal temperatures are calculated by solving Eq. (1) in the finite difference form using tridiagonal matrix algorithm, after linearization. The iterations are repeated until the temperatures are converged. The heat loss Q and improvement in heat loss per unit mass over the unfinned base surface $Q_{\mathrm{im}} = (Q - Q_{\mathrm{UB}})/m$ are calculated, where m is the mass of fin.

Results and Discussion

For all of the cases, the parameters considered are $T_B = 313$ K, $T_e = 4$ K, and $k = 177$ W/m K, and grid sensitivity studies are carried

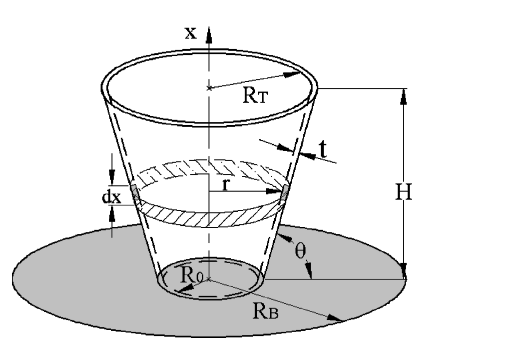

Fig. 1 Schematic of hollow conical radiating fin.

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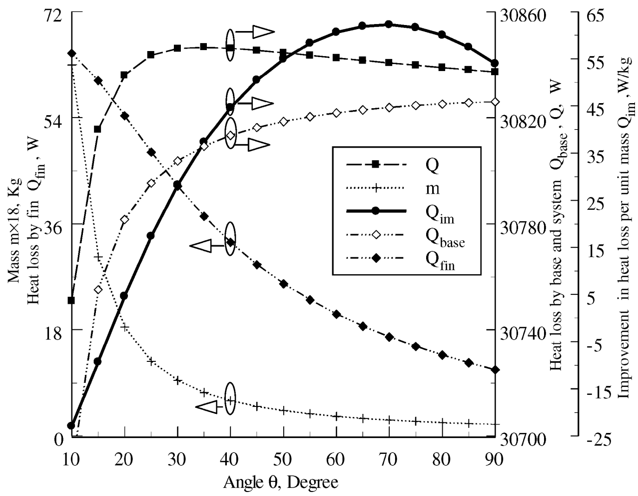

Fig. 2 Effect of fin angle on heat transfer.

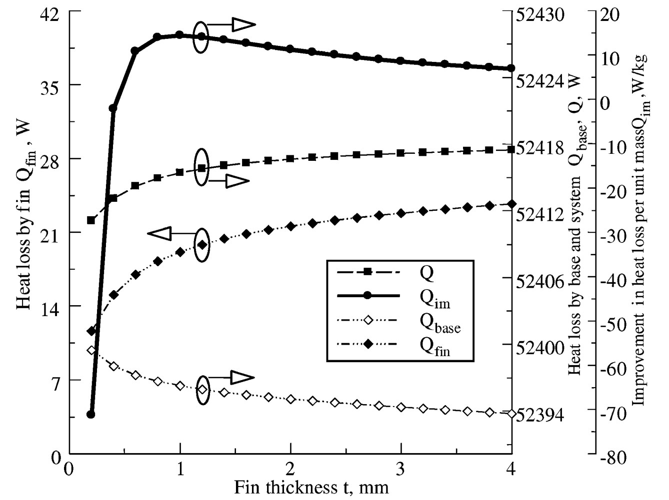

Fig. 3 Effect of fin thickness on heat transfer.

out. The analysis is validated by comparing a limiting case (hollow pin fin with inner radius tending to zero) with the analytical solution of a solid pin fin with adiabatic tip and without considering base interaction. A hollow pin fin ($\theta = 90$ deg) with an outside diameter of 4 mm, $\varepsilon = 0.85$, $H = 107.6$ mm, $R_B = 2.001$ mm, and adiabatic tip at a temperature of 300 K is considered, and the effect of inner radius on the heat loss is studied. As the inner radius tends to zero, the heat loss is obtained as 0.5596 W by extrapolation. For the corresponding solid pin fin, the heat loss is 0.5597 W based on the analytical solution,[10] the error in the present analysis being only 0.018%.

To demonstrate the usefulness of proposed geometry, a comparison is made with the solid pin fin having an outside radius of 2 mm, $H = 50$ mm, $R_B = 50$ mm, and $\varepsilon = 0.85$ by considering the base interaction. For the hollow conical fin, a constant thickness of 0.2 mm is considered, and the fin angle θ is varied such that the outer radius at the top is kept constant at 2 mm for all of the cases. The effect of θ on the ratio of improvement in heat loss per unit mass for the hollow conical fin to that of solid pin fin is studied. It is found that Q_{im} for the hollow conical fin is about 4.8 times greater than that of the solid pin fin, for an angle of about 88.5 deg.

A parametric study is carried out for a hollow conical fin with $R_0 = 62.5$ mm and $R_B = 6$ m.

Figure 2 shows the effect of fin angle for a fin with $H = 0.1$ m, $t = 1$ mm, and $\varepsilon = 0.5$. With increase in θ, the view factor from inside surface of fin to the space decreases, at the same time that from the outside surface to the space increases. With increase in θ, as the surface area of fin decreases, which is significant, the heat loss from fin alone Q_{fin} decreases, while that from the base surface Q_{base} increases because the base surface is uncovered. Hence, for lower values of θ the total heat loss from the system Q increases drastically and decreases gradually for later. This is because, for lower values of θ, the uncovering of base surface to the space is significant, and the corresponding decrease in surface area is significant for higher values. Because the mass of fin decreases with increase in θ, with the rate of decrease being steeper for lower values of θ, Q_{im} varies nonmonotonically. It is found that the optimum angle for which Q_{im} is a maximum shifts towards higher values with increase in height, emissivity and decrease in thickness, and lies in the range 55–90 deg.

Figure 3 shows the effect of fin thickness for a fin with $\theta = 70$ deg, $H = 0.1$ m, and $\varepsilon = 0.85$. Because the heat conduction into fin increases with increase in t, Q_{fin} increases monotonically, the rate being higher for lower values of t. As the temperature of fin increases with increase in t, Q_{base} reduces because of enhanced fin-base interaction. Because the rate of increase of Q_{fin} is higher compared to the corresponding reduction in Q_{base}, the total heat transfer from the system Q increases monotonically, with the rate of increase being higher for lower values of t. Because the mass of fin increases

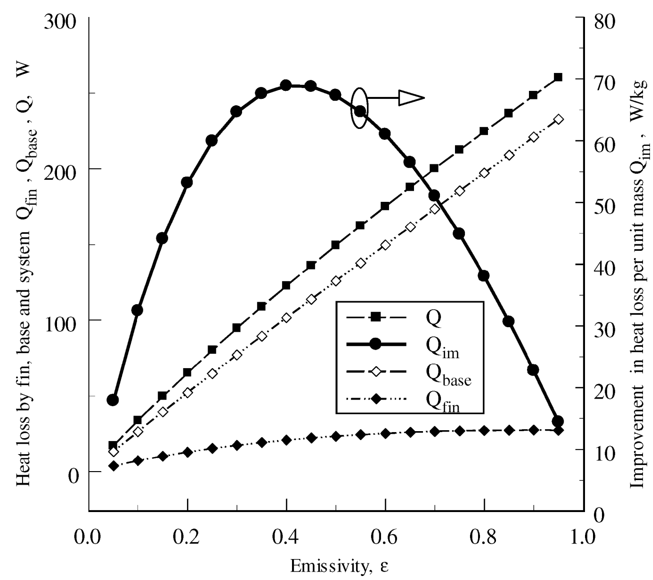

Fig. 4 Effect of emissivity on heat transfer.

linearly with increase in t, Q_{im} varies nonmonotonically. It is found that the optimum thickness for which Q_{im} is a maximum lies around 1 mm and it increases with increase in height and emissivity and decrease in angle.

It is found that Q_{im} decreases monotonically with increase in height for a given angle; hence, there is no optimum height of the fin. This could be attributed to the increase in mass of the fin with increase in height, the progression being geometric.

Figure 4 shows the effect of emissivity for a fin with $H = 0.1$ m, $\theta = 55$ deg, $t = 1$ mm, and $R_B = 0.398$ m. Because the fin-base interaction is not very significant because of relatively larger base surface considered, Q_{base} increases slightly nonlinearly with increase in ε. Further, as ε increases the absorption of radiation by the outside surface of fin increases because of gray body assumption. Hence, Q_{fin} increases nonlinearly and gradually, and the total heat transfer from fin-base surface system Q increases almost linearly. Because the heat loss by unfinned base surface increases linearly with emissivity, Q_{im} variation is nonmonotonic. It is found that the optimum emissivity for which Q_{im} is a maximum shifts towards higher values with increase in angle and thickness and decrease in height.

Finally, correlations are presented for optimum parameters and maximum Q_{im}. Based on a parametric study, Q_{im} is obtained for about 7000 data points. Of these, 596 data points for optimum angle, 118 data points for optimum thickness, and 271 data points for optimum emissivity are chosen, for which the Q_{im} is a maximum. Correlations are developed for these optimum parameters as

$$\theta_{\mathrm{opt}} = 1.398 R_0^{-0.009} H^{0.251} t^{-0.052} \varepsilon^{0.056}$$

$$t_{\mathrm{opt}} = 0.008 R_0^{0.004} H^{0.627} \theta^{-0.439} \varepsilon^{0.629}$$

$$\varepsilon_{\mathrm{opt}} = 0.869 R_0^{0.006} H^{-0.437} t^{0.264} \theta^{0.706}$$

The correlation coefficients are 0.9916, 0.9937, and 0.9916, respectively. The corresponding maximum errors are 5.7, 9.3, and 9.4%, respectively. Using 1378 data points, a correlation is developed for maximum improvement in heat loss per unit mass:

$$Q_{\mathrm{im,max}} = 0.144 R_0^{0.058} H^{-0.409} t^{-0.796} \theta^{0.291} \varepsilon^{0.444}$$

with a correlation coefficient of 0.9947 and a maximum error of 14.92%. The parameters and range for the correlations are $10 \le \theta \le 90$ deg, $0.01 \le R_0 \le 0.1$ m, $0.05 \le H \le 0.5$ m, $0.2 \le t \le 4$ mm, $0.05 \le \varepsilon \le 0.95$, $T_B = 313.15$ K, $T_e = 4$ K, $k = 177$ W/m K, and $\rho = 2770$ kg/m^3.

Conclusions

A hollow conical configuration is proposed for space radiator applications, which gives an improvement in heat loss per unit mass of about 4.8 times greater than that of the corresponding solid pin fin. It is found that there exists an optimum angle, thickness, and emissivity for which improvement in heat loss per unit mass is a maximum. The optimum angle increases with increase in height and emissivity and decreases with increase in thickness. The optimum thickness increases with increase in height and emissivity and decreases with increase in angle. The optimum emissivity increases with increase in angle and thickness and decreases with increase in height. Correlations are presented for optimum angle, thickness, emissivity, and corresponding maximum improvement in heat loss per unit mass.

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Radiation with Mixed Convection in an Absorbing, Emitting, and Anisotropic Scattering Medium

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Nomenclature

f	=	dimensionless stream function
g	=	acceleration of gravity
h	=	heat-transfer coefficient
I	=	dimensionless radiation intensity
k	=	thermal conductivity
N	=	conduction-radiation parameter
Nu_x	=	local Nusselt number
$P_n(\mu)$	=	Legendre polynomial of the first kind of degree n
Pr	=	Prandtl number
$p(\mu, \mu')$	=	phase function
Q^r	=	dimensionless radiative heat flux
q^r	=	radiative heat flux
Re_x	=	local Reynolds number
T	=	temperature
x, y	=	physical coordinates along and normal to the wall
β	=	volumetric coefficient of thermal expansion
β_0	=	extinction coefficient
ε	=	emissivity of plate surface
η	=	nonsimilarity variable
θ	=	dimensionless temperature
λ	=	coefficient of the thermal expansion
μ	=	direction cosine
ν	=	kinematic viscosity
ξ	=	nonsimilarity variable
ρ	=	fluid density
ρ_d	=	diffuse reflectivity of plate surface
$\bar{\sigma}$	=	Stefan–Boltzmann constant
τ	=	optical variable, $= \xi\eta$
φ	=	stream function
Ω	=	buoyancy parameter
ω	=	scattering albedo

Subscripts

w	=	wall
∞	=	external flow

Introduction

THE problem of radiation on mixed convection along a vertical plate with uniform surface temperature has been studied theoretically because the radiation effects on the mixed convection flow are important in the context of space and processes involving high temperature. At high temperatures, thermal radiation can significantly affect the heat transfer and temperature distribution in boundary-layer flow of a participating fluid. Cess,[1] Arpaci,[2] Soundalgekar and Takhar,[3] and Hossain and Takhar[4] have utilized the optically thin limit and the optically thick limit approximation methods for these studies. But these approximations are accurate

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